

Likelihood ratios, predictive values, and post-test probabilities

The purpose of this quick tutorial is to help you better understand how to use likelihood ratios (LRs), predictive values (PVs), and post-test probabilities in medical writing. The key point is that LRs are calculated differently depending on whether a test has only two possible results (dichotomous), or whether it has more than two possible outcomes (multichotomous). Examples of the latter include a clinical decision rule (CDR) that identifies low, moderate and high-risk groups, or reporting serum ferritin as < 20, 20 – 60, 61 – 100, and > 100 mg/dl.

Dichotomous tests

A likelihood ratio (LR) for a dichotomous test is defined as the likelihood of a test result in patients with the disease divided by the likelihood of the test result in patients without the disease. For example, the likelihood of abnormal fecal immunochemical test (FIT) in patients with colorectal cancer, divided by the likelihood of an abnormal FIT in those without cancer. A LR close to 1 means that the test result does not change the likelihood of disease or the outcome of interest appreciably. The more the likelihood ratio for a positive test (LR+) is greater than 1, the more likely the disease or outcome. The more a likelihood ratio for a negative test is less than 1, the less likely the disease or outcome. Thus, LRs correspond nicely to the clinical concepts of ruling in and ruling out disease.

One suggested way to interpret LR is as follows (Ebell, 2005 Information Mastery AAFP Home Study):

- > 10 Large and often conclusive increase in the likelihood of disease
- 5 – 10 Moderate increase in the likelihood of disease
- 2 – 5 Small increase in the likelihood of disease
- 1 – 2 Minimal increase in the likelihood of disease
- 1 No change in the likelihood of disease
- 0.5 - 1.0 Minimal decrease in the likelihood of disease
- 0.2 - 0.5 Small decrease in the likelihood of disease
- 0.1 - 0.2 Moderate decrease in the likelihood of disease
- < 0.1 Large and often conclusive decrease in the likelihood of disease

The following example calculates the LR+ and LR- using sensitivity and specificity:

	Gold std for flu +	Gold std for flu -		Formulas
Flu test +	a. 70	b. 10	a + b 80	Sensitivity = $a / (a+c) = 70/100 = 70\% = 0.70$
Flu test -	c. 30	d. 90	c + d 120	Specificity = $d / (b+d) = 90/100 = 90\% = 0.90$
	a + c 100	b + d 100		LR+ = $\text{sens} / (1-\text{spec}) = 0.70 / (1 - 0.90) = 7.0$
				LR- = $(1-\text{sens})/\text{spec} = (1 - 0.70) / 0.90 = 0.33$

So, the flu test is moderately to very good at ruling in flu when positive (LR+ 7.0), but is less helpful when negative at ruling out disease (LR- 0.33). That is because the LR- is one-third, while the LR+ is much greater than its inverse of 3 (i.e. 7). Many tests are “asymmetric” in this way, and are better at ruling in than ruling out, or vice versa.

For a dichotomous test with two possible outcomes, the positive predictive value is the likelihood that a patient with a positive test actually has flu, and the negative predictive value is the likelihood that a patient with a negative test does not have flu. The post-test probability of flu is the probability of flu given a positive or negative test. The latter is different and I think a bit more useful.

$$\text{Predictive value +} = a / (a+b) = 70 / 80 = 87.5\% = 0.875$$

$$\text{Predictive value -} = d / (c+d) = 90 / 120 = 75\% = 0.75$$

$$\text{Post-test probability for positive test} = a / (a+b) = 70 / 80 = 87.5\% = 0.875$$

$$\text{Post-test probability for negative test} = c / (c+d) = 30 / 120 = 25\% = 0.25$$

So, as we expect from the likelihood ratios for this test, given a “starting point” of 50% for the overall prevalence of flu, a positive test increases the probability of flu to 88% (an absolute increase of 38% [88-50]). A negative test, however, still leaves a 25% probability of flu (an absolute decrease of 25% [50-25]).

Multichotomous tests

When there are 3 or more possible outcomes for a test or CDR, we refer to the test as having multiple strata (risk groups). As noted above, these tests are “multichotomous” rather than “dichotomous”. This is common with CDRs that often divide patients into multiple risk groups such as low, moderate and high risk. It can also apply to other kinds of tests like lab results such as serum ferritin < 20 ng/ml, 21 – 80 ng/ml, and > 80 ng/ml. The advantage of doing this is that it provides richer information and is more useful for clinical decision-making, as opposed to categorizing everyone as simply positive or negative. Therefore, we should not collapse risk strata, for example combining low and moderate risk groups and comparing that with high risk, as that loses information. Below is a table showing how to calculate likelihood ratios with 3 or more risk groups or outcomes. Note that each risk stratum or risk group has its own likelihood ratio, and there is no such thing as an overall LR+ or LR- anymore.

Risk group	Positive	Negative	LR
Low risk	a	x	$(a / T_{pos}) / (x / T_{neg}) = LR_{low}$
Moderate risk	b	y	$(b / T_{pos}) / (y / T_{neg}) = LR_{moderate}$
High risk	c	z	$(c / T_{pos}) / (z / T_{neg}) = LR_{high}$
	T_{pos}	T_{neg}	

The likelihood ratio for each stratum is calculated as the likelihood of that test result in patients with a positive test divided by the likelihood of that result in patients with a negative test.

The predictive value for each stratum is just positive test results / total patients in each stratum or risk group. A worked example from a study of the CRB-65 clinical decision rule (<https://www.aafp.org/afp/2006/0415/p1425.html>) for predicting mortality in patients with pneumonia is shown below:

CRB-65 risk group for patients with pneumonia	Deaths	Survivors	LR	Predictive Value
Low risk	3	125	$(3/87) / (125/929) = 0.25$	$= 3 / (3+125) = 2.3\%$
Moderate risk	58	725	$(58/87) / (725/929) = 0.85$	$= 58 / (58+725) = 7.4\%$
High risk	26	79	$(26/87) / (79/929) = 3.5$	$= 26 / (26+79) = 24.8\%$
	87	929		

In this population, the low risk and high risk groups had significantly lower and higher likelihoods of mortality, respectively. The likelihood ratios can also be used to calculate stratum-specific predictive values given any baseline probability of disease. In the sample tables below, we use the likelihood ratios to do just that. The advantage is that we are not just stuck with the prevalence from the original study, but can extrapolate that to more typical prevalences found in different practice settings or populations. Below, we calculate the probability of pneumonia in 2 settings, using a baseline risk of pneumonia for each setting derived from large cohort studies. The same approach is shown for strep throat, calculating probabilities of streptococcal infection using different baseline prevalences for adults and children from the literature.

Sample Table for Multichotomous Test (3+ outcome categories)

Risk score	Likelihood Ratio	% with community acquired pneumonia given baseline prevalence of:	
		4% (primary care)	20% (ED)
-3	0.00	0.0%	0.0%
-2	0.27	1.1%	6.3%
-1	0.59	2.4%	12.9%
0	0.82	3.3%	17.1%

1	3.6	13.0%	47.4%
2	4.3	15.2%	51.8%
3	12	34.1%	75.6%
4	14	36.8%	77.7%
5	9.3	27.9%	69.9%

Sample Table for Dichotomous Test (2 outcome categories)

Sign or symptom	LR+	LR-	% strep given baseline prevalence of 10% (adult)		% strep given baseline prevalence of 30% (child)	
			Finding Present	Finding Absent	Finding Present	Finding Absent
Tonsillar exudate	5.0	0.2	30%	3%	70%	6%
Cervical adenopathy	3.0	0.9	20%	8%	50%	25%
Etc.						